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EXCELLENCE PROGRAM-SYJC (COMMERCE), 2019-20
SYNOPSIS
MATHEMATICS AND STATISTICS - PART 1

CONTUNUTY

[08 MARKS FOR H.S.C.]

❖ LIMITS :-

1. $\lim_{x \rightarrow a} P(x) = P(a)$

2. $\lim_{x \rightarrow a} \sin x = \sin a$

3. $\lim_{x \rightarrow a} \cos x = \cos a$

4. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\sin x}$

5. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\tan x}$

6. $\lim_{x \rightarrow 0} \cos x = 1$

7. $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$

8. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$

9. $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$

10. $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$

1) CONTINUTY OF A FUNCTION AT A POINT:

A function is said to be continuous at a point $x = a$ if

- a) $\lim_{x \rightarrow a} f(x) = f(a)$ and
- b) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$

Note: The following functions are always continuous on their domain

- i) Constant function ie $f(x) = k, k \in \mathbb{R}$ is constsnt.
- ii) Polynomial function ie $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$.
- iii) Rational function ie $\frac{f}{g}$, f & g are polynomial functions.
- iv) Trigonometric function ie $\sin(ax + b), \cos(ax + b)$, $a \& b \in \mathbb{R}$
- v) Exponential function ie $f(x) = a^x$ or $f(x) = e^x, a > 0, a \neq 1$
- vi) Logarithmic function ie $f(x) = \log_a x, a > 0, a \neq 1$

2) ALGEBRA OF CONTINUOUS FUNCTIONS

If f and g are two real valued functions defined on the same domain and continuous at $x=a$, then

- i) The Function kf is continuous at $x = a, k \in \mathbb{R}$ is constsnt
- ii) The Function $f \pm g$ is continuous at $x = a$
- iii) The Function $f \cdot g$ is continuous at $x = a$
- iv) The Function $\frac{f}{g}$ is continuous at $x = a, g(a) \neq 0$.
- v) Composite functions $f[g(x)]$ and $g[f(x)]$ are continuous at $x = a$

3) TYPES OF DISCONTINUITY:

➤ REMOVABLE DISCONTINUITY:

A real valued functions f is said to have **removable discontinuity** at $x = a$ if

i) $\lim_{x \rightarrow a} f(x) \neq f(a)$ or

ii) $\left[\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a} f(x) \right] \neq f(a)$

This type of discontinuity can be removed by redefining the function f at $x = a$ as

$\lim_{x \rightarrow a} f(x) = f(a)$ to make continuous function.

➤ IRREMOVABLE DISCONTINUITY:

A real valued functions f is said to have **irremovable discontinuity** at $x = a$

i) $\lim_{x \rightarrow a} f(x)$ does not exist or

ii) $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$ or

iii) $\lim_{x \rightarrow a^-} f(x)$ does not exist or $\lim_{x \rightarrow a^+} f(x)$ does not exist

Such a function cannot be redefined.